XV. *On the Law of the Resistance of the Air to Rifled Projectiles.*

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AT the beginning of this month Lieut.-Colonel H. R. HALFORD applied to me to obtain for him the law of atmospheric resistance resulting from his experiments in shooting with METFORD'S match-rifle, a small-bore with increasing pitch. I found the resistance to vary as the cube of the velocity.

As the calculations and experiments were all made without any notion of the resulting law, and without any knowledge of the work already done by Professor HÉLIE and Professor BASHFORTH, they afford a remarkable confirmation of the results obtained by those gentlemen. This is the more worthy of notice, as their data belong to pieces of large calibre, and mine to small-bore bullets.

The experimental data originally submitted to me were simply the elevations found necessary to give the different ranges of every 100 yards from 100 to 1100. These were as follows:—

Yards range.	Elevation. ° ′ ″
100	9 30
200	20 —
300	31 30
400	44 —
500	57 30
600	1 12 —
700	1 27 30
800	1 44 —
900	2 1 30
1000	2 20 —
1100	2 39 30

I was informed that these were the means derived from a great number of selected trials, bad shots being of course discarded.

The laws of the motion were arrived at tentatively. I began the calculations from an assumed velocity of 1360 feet per second, which is seen by the shorter ranges to be very nearly true, of necessity. I then found that the law of the squares did not fit the results at all, but that the law of the cubes very nearly did so, and a slight alteration in the assumed velocity made them do so with exactness.

The method used was to determine the coefficient of resistance m, in $\frac{dv}{dt}=-mv^3$, which

gives $m=\frac{2(tv_1\cos\theta-s)}{s^2v_1\cos\theta}$, v_1 being the initial velocity, s the actual range, and θ the inclination. The time t was that of the range *in vacuo* due to the supposed initial velocity and given elevation. It was assumed, *first*, that the vacuum time might be taken to represent the real time, and, *second*, that the horizontal resistance might be taken to vary as the cube of the horizontal velocity. The result is given in the following Table.

Calculation of Resistance, supposed to vary as the Cube of the Velocity.

Initial velocity taken as 1358 feet per second.

Angle of elevation.	Actual range, in feet.	Range *in vacuo*, in feet.	Reduction of range by resistance, in feet.	Time of flight *in vacuo*, in seconds.	Coefficient of resistance proportionate to	Departure from mean value.	Final velocity, in feet per second.	Velocity lost in each 100 yards, in feet per second.	Actual range, in yards.
° ′ ″ 9 30	300	316·62	16·62	0·23316	16·620	−0·018	1222·4	135·6	100
20	600	666·57	66·57	0·49085	16·643	+0·005	1111·5	110·9	200
31 30	900	1049·81	149·81	0·77309	16·647	+0·009	1018·8	92·7	300
44	1200	1466·32	266·32	1·07985	16·647	+0·009	940·6	78·2	400
57 30	1500	1916·02	416·02	1·41115	16·643	+0·005	873·4	67·2	500
1 12	1800	2399·00	599·00	1·76695	16·654	+0·016	815·2	58·2	600
1 27 30	2100	2915·04	815·04	2·14732	16·644	+0·006	764·3	50·9	700
1 44	2400	3464·11	1064·11	2·55206	16·634	−0·004	719·4	44·9	800
2 1 30	2700	4046·12	1346·12	2·98133	16·639	+0·001	679·4	40·0	900
2 20	3000	4660·92	1660·92	3·43504	16·633	−0·005	643·7	35·7	1000
2 39 30	3300	5308·37	2008·37	3·91317	16·616	−0·022	611·5	32·2	1100
These two columns contain the data.				Mean	16·638				

Resulting Coefficient of resistance $=0{\cdot}00000\ \ 02723$.

The coefficient of resistance, so determined for each range, has no sensible variation, thus proving that the facts are exactly met by the law that the resistance varies according to the cube of the velocity, with a coefficient of resistance, for this particular bullet, of 0·00000 02723. This is subject to the assumptions previously mentioned. It is further either involved or assumed that the bullet presents an invariable aspect in a horizontal direction.

These assumptions are sufficiently right for such a very low trajectory as is given by 2° 40′ for 1100 yards, giving a vacuum height of 64 feet. I got out a graphical solution of the resisted time of a bullet thrown with a velocity of 64 feet per second (which corresponds to that height) and with a coefficient of 0·000001, and I found the variation in time too small to detect in fine curves carefully drawn with ordinates of nearly 2 feet in length. I consider this as amounting to proof that the difference between real and vacuum time is insignificant in this problem. The second assumption, as to the horizontal resistance varying as the cube of the horizontal velocity, is justified by the low trajectory. On the question of aspect, my present knowledge does not authorize me to form any decided opinion.

The graphical method mentioned above was devised to evade the arithmetical difficulties of the question; and I hope shortly to be able to publish an account of it*.

Colonel HALFORD has since been so obliging as to communicate to me the following notes of the experimental data. These are as follows:—

" *The means by which the angles of* METFORD'S *match-rifles were arrived at.*

"The rifles were shot at $12\frac{1}{2}$ yards to get the zero on the tangent-scale.

"This distance is taken because, with a radius of $12\frac{1}{2}$ yards, $\frac{1}{8}$ of an inch is one minute of angular space, an easy quantity to deal with. Corrections made for height of fore sight above axis of bore, and for fall of bullet, at $12\frac{1}{2}$ yards, taken as one minute.

"The tangent-scale is then carefully cut for radius between the sights to read to minutes with a vernier. It can be read to $15''$ by careful inspection. The Table of angles is then made from careful registers of shooting in all weathers, but especially calm weather, at different distances, but chiefly at 200 yards, 500, 600, 800, 900, and 1100 yards. Nine times out of ten the angles will be found to be perfectly correct in calm weather. Difference of barometrical pressure will make a slight difference. The Table furnished [*i.e.* of elevations corresponding to ranges] was drawn out by Mr. METFORD, and has been very heavily tested by myself, and tallies exactly with my experience. The angles have been compared with those used by other men using similar rifles, and have been found to tally, making slight allowance for the little differences of speed which different barrels will give—usually not more than is due to $0°\ 2'$ of elevation in 1000 yards. I must have fired from 8000 to 9000 shots. I have most of my shots registered. I should say that the Tables have been worked out from results given by at least 30,000 shots.

Ranges.	Elevations, Table No. 1.	Elevations, Table No. 2.	Elevations, Table No. 3.
Yards.	° ′ ″	° ′ ″	° ′ ″
100	9 15	9 30	10 30
200	19 30	20	20 30
300	30 45	31 30	32 30
400	43	44	45 15
500	56 15	57 30	59
600	1 10 30	1 12	1 14
700	1 25 45	1 27 30	1 29 30
800	1 42	1 44	1 46 30
900	1 59 30	2 1 30	2 4 45
1000	2 17 30	2 20	2 23
1100	2 36 45	2 39 30	2 42 30

"No. 3 Table is not perfect in itself; it is a Table used occasionally with a head wind, and has not been so carefully worked out as the others; a head wind is so disturbing in its effects that, practically, it is almost impossible to verify it. The other Tables are reliable within very small fractions.

* [This has since appeared in the Philosophical Magazine for June 1868.]

"Details of bullet, &c.

"METFORD'S match-rifle: *calibre* 0·4605 inch.

"*Charge*: 90 grains, CURTIS and HARVEY'S No. 5 powder.

"*Length of bullet* . . .	1·45	inch.
"*Diameter* at base . . .	0·451	,,
$\frac{1}{2}$ inch from base	0·45	,,
$\frac{7}{8}$ inch from base	0·447	,,

"*Weight*: 530 grains.

"*Material*: lead hardened with alloy.

"*Shape*: cup-shaped in rear, nearly cylindrical for the first inch, and gradually rounding off from there to a spherical or paraboloidal head, not pointed."

These notes were not given to me until after my calculations had been made. It will be observed that the data originally supplied, and from which I worked, are those of Table 2. This appears to be the most reliable, being very near the mean of the other two.

On the whole, I consider myself singularly fortunate in having had such a very good series of experiments from which to calculate.

It will not fail to be noticed that the velocity of sound is intermediate between the initial and final velocities of the bullets at the longer ranges.

Note added 3rd August 1868.—For the sake of comparison with other results, I give the reduction of this coefficient of resistance to the standard of one pound weight avoirdupois, and one inch transverse section, on the (doubtful) assumption that the coefficient varies inversely as the weight, and directly as the transverse section.

Coefficient reduced to 1 lb. weight and to one square inch section 0·00000 01267,

Ditto, and to one circular inch section . . . . 0·00000 00995.